The semantics of minimal requirements

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A maths problem — "One-half of a road construction project was completed by 6 workers in 12 days. What is the smallest number of workers needed to finish the rest of the project in 4 days?"[1]

This problem asks after the number of workers that are *minimally required* to finish the project in 4 days. Expressions of minimal requirement are common and interpreted without problems. Nevertheless, this paper will show that the standard semantics for modals fails to deliver the correct interpretation.

The puzzle — In the context of the above maths problem, the following is a true statement.

(1) To finish the project in 4 days, one needs 18 workers.

The sentence in (1) is predicted to be true by the theory of von Fintel and Iatridou 2005. In their account, *need* receives the standard analysis of a universal quantifier over worlds. *To p, need to q* is true in w iff q is true all the worlds in the modal base of w where p is true: $[\![\text{to } p\text{, need to } q]\!]^w = 1 \Leftrightarrow \forall w'[w' \in \cap f(w)\ \&\ p(w') \rightarrow q(w')]$. Given what we know about the productivity of the workers, we come to expect that worlds in which the project is finished in 4 days are worlds in which 18 workers are involved. Now consider, (2).

(2) To finish the project in 4 days, one needs workers.

The theory of von Fintel & Iatridou predicts, correctly I believe, that (2) is true (though not very informative). There are no worlds in which the project is finished in 4 days where no worker is involved; *all* those worlds have *at least one worker* working on the project. Now let us turn to minimal requirements. Intuitively, (3) is true (and is a way of expressing the answer to the maths problem).

(3) The smallest number of workers needed to finish the project in 4 days is 18.

This is true, on an intuitive level, because using 18 workers is (part of) *one* of the ways of reaching the goal, and, in fact, it is the way which uses the minimal number of workers. (That is, more workers also allow you to finish in 4 days, fewer don't allow you to do so). Theoretically, however, it is completely unclear how we arrive at such an interpretation. The problem is this: given the fact that (2) is true, one would expect (3) to be false. Why? Because (2) suggest that *the smallest number of workers needed to finish the project in 4 days is 1.* Formally, we expect the maths problem above to ask after the number described in (4). In the context of the maths problem, this number equals 1.

(4) the smallest n such that $\exists x[\#x = n\ \&\ \text{worker}(x)\ \&\ \text{work\_on\_project}(x)]$ is true in all maths problem worlds in which the project is finished in 4 days

One might think that the puzzle is merely due to the fact that (4) is not about the *exact* number of workers that were involved in the project, but rather how many there were *at least*. In other words, (4) takes for granted a weak semantics for numerals, and ignores potential stronger readings. So, maybe if we change the formula in (4), which stands for *there are at least n workers on the project*, to something which represents *there are exactly n workers on the project*, the puzzle disappears. Alas, such a change would yield undefinedness: there is no n such that exactly n workers were working on the project in every maths problem world in which the project is finished in 4 days. The reason is that there is no unique

[1] `https://mathcounts.org/Page.aspx?pid=1284`

number of workers that allows you to finish in 4 days: 18 will do, but so will 19 and 20, etc.

In general, the standard theory predicts that if S is an entailment scale of propositions, and p is a proposition on this scale, then if p is a minimal requirement for some goal proposition q, then a statement of the form "the minimum requirement to q is p" is always predicted to be false, except when p is the minimal proposition of S. This predicts that minimal requirements could never be expressed, since they would always correspond to the absolute minimum, and therefore never be informative.

The INUS condition in *need* — In reaction to a (non-related) problem noticed by Nissenbaum (2005), von Fintel & Iatridou add an INUS condition to the semantics of *need*. An INUS (Mackie 1965) is an **i**nsufficient but **n**onredundant part of an **u**nnecessary condition which by itself is **s**ufficient: to p, need q, entails that there exists an unnecessary yet sufficient condition of which q is an insufficient yet nonredundant part. Formally, $\exists P[P+q \Rightarrow p \ \& \ P \not\Rightarrow q]$.[2] In the maths problem example, there being exactly 18 workers is an INUS for finishing the project in 4 days, and so is there being exactly 19 workers, there being exactly 20 workers, etc. I propose that the INUS condition plays a pivotal part in examples like (3). The INUS condition is what minimal requirements are all about. In the example above, 18 is the smallest number for which the INUS condition holds. That is, there is no way P of finishing the project in 4 days of which having a group of 17 (or fewer) workers is an essential part. That is, the only propositions which together with the proposition that there are 17 workers entails that the project is finished in 4 days are propositions which themselves entail that the project is finished in that time (e.g. there being 18 workers).[3]

Non-universal *needs* — Adding the INUS condition to the standard ∀-semantics of *need* will not do, however, to account for (3). This is because 18 is the *only* number such that both (i) there is a group of so many workers in every goal world and (ii) having so many workers is an INUS for finishing the project in 4 days. If *need* comprises both these conditions, one would expect (3) to be infelicitous, since it applies a singular superlative to a singleton set. (Cf. *#the tallest current king of Spain*.) The upshot is that a semantics involving universal quantification over possible worlds is too strong for such examples. In fact, there are plenty of other examples with modals like *need* in which a weaker form of quantification is called for. The statement in (5-a) is compatible with the queen also meeting golden medal winners and (5-b) does not (necessarily) say that people who answer all questions correctly fail the test.

(5) a. To meet the queen, you need to win a silver medal.
b. To pass the test, you have to answer almost all questions correctly. (Nouwen 2006)

Two directions — The entailment patterns that come with the INUS condition are antithetical to the patterns that come with universal quantification over worlds. If all accessible p-worlds are q-worlds, then for any q' that is entailed by q it holds that all accessible p-worlds are q'-worlds. This is why (2) follows from (1). On the other hand, if q is an essential way of achieving p, then any q' entailing q is also an essential way of achieving p. This accounts for why (3) is the correct answer to the maths problem.

The data show that these two principally incompatible directions of inference are available for the same lexical item *need*. I propose therefore that modals like *need* can be used to express universal quantification over worlds and INUS conditions separately. Under normal circumstances, however, these two senses of *need* will be exhaustified. On its first sense, *needs* q then states that q is the strongest proposition that is true in every accessible goal world. On its second sense, *need* q states that q is the weakest (or easiest/best) proposition that is an essential part of achieving the goal. Under normal circumstances, then, the two senses of *need* will be indistinguishable.[4] (Example (1) is an example of this.) It is only

[2]This presentation is based on (von Fintel and Iatridou 2005).

[3]For illustration, "there are 17 workers" + "there is 1 worker" does not entail that there are 18 workers, independent of how we interpret the numerals.

[4]This is reminiscent of and was inspired by Schwager 2005, where a similarly exhaustified existential semantics was proposed for certain imperatives, which are normally associated with necessity.

when *need* is combined with scalar operations that the two polarly opposed versions become visible.

Minimal requirement is always minimal *teleological* requirement — An important point to make is that I think that the puzzle I have presented is limited to teleological modality only. That is, the problematic examples involve the sufficient or necessary conditions for reaching a goal. There is no similar puzzle with epistemic statements. In other words, I do not believe that the standard approach to strong modals is wrong; I just think that it is the wrong analysis of teleological modality.

To show that the problems I have introduced are limited to goal oriented modality, consider the following example: (6-b) is not a way to express a lower bound on what the speaker knows.

(6) I've seen you put 4 marbles in the box, but I don't know how many there were in the box to begin with. #So, the smallest number of marbles that must be in the box is 4.

This example contrasts with (3) above, yet the context is structurally the same. For instance, the epistemic state of the speaker of (6) could be described as follows.

(7) a. In some but not all compatible worlds: there are (at least) 5 marbles in the box.
b. In all compatible worlds: there are (at least) 4 marbles in the box.
c. In all compatible worlds: there are (at least) 3 marbles in the box.
d. In all compatible worlds: there are marbles in the box.
e. $\Longrightarrow$ the smallest number of marbles that must be in the box is 1

On the basis of similarly structured situations, we could express a minimal requirement in the teleological case, but apparently this is not possible in the epistemic case. It appears then that the standard theory actually makes the right predictions in non-teleological contexts. It predicts correctly that (6) could never be a true statement, and that it is therefore infelicitous. The upshot is that minimal requirement is a strictly teleological notion.

Application: Modified numerals — The proposal has consequences for theories of modified numerals. As argued in Nouwen 2009, the semantics of (8) is equivalent to that of (3).

(8) To finish the project in 4 days, you need { minimally / at least } 18 workers.

In previous accounts (Geurts and Nouwen 2007, Krifka 2007) of the semantics of the numeral modifiers like *minimally* and *at least*, examples like (8) were interpreted using an independent mechanism for the interaction of modifier and modality. The paper shows that these data are part of a much bigger picture involving teleological modality and scales.

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